Wufi Hassan P1

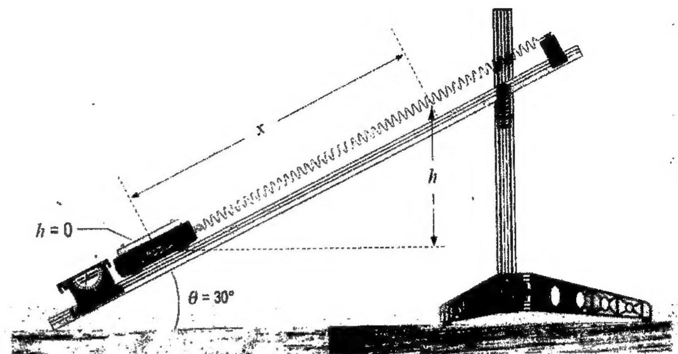
Investigation 10C: Springs and the conservation of energy

Essential question: Can energy conservation be used to predict the behavior of a system?

A system consisting of a cart on an inclined track, attached to a spring, has three forms of mechanical energy as the cart rolls down the track. By carefully modeling the flow of energy between potential, elastic, and kinetic we can accurately predict how far the cart will roll before it is stopped by the spring.

Part 1: Build the mathematical model

1. Imagine a cart on an inclined track (30°), attached to a spring, similar to the picture.
2. The cart is released from rest at the top of the track (spring is unstretched) and rolls freely down until the spring stops it after stretching some distance.



Questions

a. Write equations for the total energy of the cart and spring at the top of the track (before the cart is released), and the total energy at the bottom of the track (when the spring stops the cart).

$Energy_{total} = 0 + (m)(9.8)h + 0 = 9.8mh$ | $Energy_{total} = \frac{1}{2}kx^2$

b. Use your equations to build a mathematical model for the maximum distance the cart travels down the track in terms of mass *m* and spring constant *k*.
NOTE: Because the track is inclined 30°, as the cart travels a distance x down the track the vertical height through which it travels is x/2, or h = x/2.

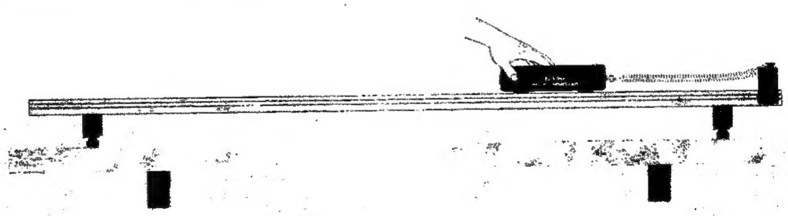
$$9.8m\frac{x}{2} = \frac{1}{2}kx^2$$

$$\Rightarrow 19.6m = x^2 \cdot \frac{2}{x}k \quad \therefore x = \frac{19.6m}{2k}$$

$$\Rightarrow 2xk = 19.6m$$

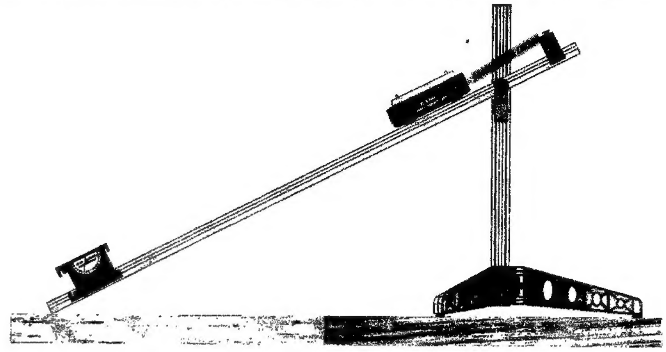
Part 2: Determine spring constant

1. Set up your equipment like the picture using the stiffest spring. Make sure the track is level.



2. Open the **10C_SpringConstant** experiment file, and then power-on your Smart Cart and connect it using Bluetooth.
3. In your software, zero the Smart Cart force sensor while nothing is touching the hook.
4. Attach the spring to the hook on the cart. Position the cart so that it is about to place the spring under tension.
5. Record data as you slowly pull the cart about 70 cm along the track. Data collection will end automatically after 4 s.

6. Find the slope of the straight line portion of the graph—this is the spring constant for the spring. Record the spring constant value k, and save your data for reference later in the investigation.



Spring constant k = 6.44

Part 3: Predict the maximum distance

1. Set up your equipment like the picture.
2. Measure the mass of the Smart Cart plus two 250-g cart masses.

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Mass m = .~~98~~85

3. Use the mathematical model for maximum distance derived in Part 1, and your measured values for m and k to predict the distance the cart will roll down the track before being stopped by the spring.

0.766

Maximum distance d_{theo} = ~~[illegible]~~ m
(theoretical)

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$$\frac{(.\cancel{98}85)(19.6)}{2(6.44)}$$

4. Open the **10C_EnergyTransformations** experiment file, and then reconnect your Smart Cart.

5. Hold the cart on the track so that the spring is unstreched.

6. Start data collection, release the cart. Catch the cart at the top of its rebound. Data collection stops automatically after 3 s.

7. Use the tools in your graph to determine the maximum distance the cart travelled.

Maximum distance d_{exp} = 0.78 m
(experimental)

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8. Use your measured values and motion data to complete the table below.

Table: Motion and energy data for a cart and spring system on a 30°-inclined track

Position	Spring extension x (m)	Cart speed v (m/s)	Cart height $h = (d_{exp} - x)/2$ (m)	Cart potential energy E_p (J)	Cart kinetic energy E_k (J)	Spring potential energy E_s (J)	Total energy E_T (J)
When the cart is released	0	0	0.39	1.905	0 J	0 J	
At maximum speed down	0.37	1.34	0.205	1.0	0.448		
At maximum distance	0.78	0	0	0	0 J	1.905 J	
At the top of the rebound	0	0	0.39	1.9050	0 J	0 J	

Questions

a. How do your theoretical and experimental values for maximum distance compare? What may have caused differences between them?

b. How did the total energy of the cart and spring system change as the cart rolled down the track? Explain your answer.

c. Create a bar chart to display the kinetic, gravitational potential, elastic potential and total energies at each of the following locations: immediately after being released; when the cart reaches its maximum speed down the track; when the cart reaches its lowest point; and at the top of the rebound. What patterns do you see?

d. Calculate the efficiency of the system by using data from the table for two scenarios:
(i) for the trip from the starting point to the lowest point reached; and
(ii) for the trip from the starting point to the final rebound position.

$$Efficiency = \frac{\text{final energy}}{\text{initial energy}} \times 100\%$$

e. How can the difference between the two efficiencies be explained?
Hint: Look at the graph of force vs spring extension that you used to determine the spring constant early in this investigation.